

Lecture 10

Recap of LSI, Fourier, and Sampling

10.1 Learning Objectives

- Recall the major concepts from the first few weeks of the course.

10.2 Introduction

In this lecture, we will go over some of the topics covered during the first 10 lectures, including:

- Linear Shift Invariant (LSI) systems,
- Fourier analysis,
- Sampling,

where each of these topics was covered first in the 1D case and then extended to the multi-dimensional case.

Question: Why do we often call our linear shift invariant systems 'filters'?

Question: In a radiation therapy planning scenario, if we assume that our tissue is spatially heterogeneous, is it likely that we will be able to express our distribution of absorbed dose $d(\mathbf{r})$ as a convolution between the distribution of primary interactions $A(\mathbf{r})$ and a dose kernel $K(\mathbf{r})$, ie: $d(\mathbf{r}) = A(\mathbf{r}) * K(\mathbf{r})$?

10.3 One of the Homework Questions: Magnitude vs Phase

In Matlab (or Python), load the MAT-file 'dogcat.mat', which contains 2 images ('im1' and 'im2'). Calculate the 2D (discrete) Fourier Transform of each of these images (eg: using the command `fft2`, or see examples in our Jupyter notebooks).

- A. Create a ‘mixed’ image by using the magnitude from the Fourier Transform of im1, and the phase from the Fourier Transform of im2, and then performing a 2D (discrete) inverse Fourier Transform (eg: using the command `ifftn`). Does this ‘mixed’ image look more like im1 or im2?
- B. Create another ‘mixed’ image by using the magnitude from the Fourier Transform of im2, and the phase from the Fourier Transform of im1, and then performing a 2D (discrete) inverse Fourier Transform (eg: using the command `ifftn`). Does this ‘mixed’ image look more like im1 or im2?

What does this little experiment suggest about the relative importance of the magnitude vs the phase of the Fourier Transform of an image?

SOLUTION

Below is some sample Matlab code to solve this exercise. See the results in Figure 10.1. Note that the phase of the FT has a lot more influence on the image appearance.

One intuitive interpretation: This dominance of the phase can be interpreted as due to the fact that most natural images (eg: images of cats and dogs) have Fourier Transforms with pretty similar magnitude distribution, ie: the magnitude of the signal is distributed in a similar decreasing fashion from low frequencies to high frequencies. However, the phase of the Fourier Transforms, which determines how the different frequency components are aligned with respect to each other (eg: where the image edges appear), is quite different from one natural image to another.

```
% Load images
load dogcat.mat

% Apply FT to each image
fim1 = fftn(im1);
fim2 = fftn(im2);

% Create our 'mixed' images
fim3 = abs(fim1).*exp(i*angle(fim2));
fim4 = abs(fim2).*exp(i*angle(fim1));
im3 = ifftn(fim3);
im4 = ifftn(fim4);

% Display images
imagesc(abs([im1 im2; im3 im4]));axis equal tight off;colormap gray
```

10.4 Misfocused Camera

Optical imaging systems (eg: cameras) can often be described in terms of the ‘Optical Transfer Function’ (OTF), which is simply the the impulse response of the camera, in the

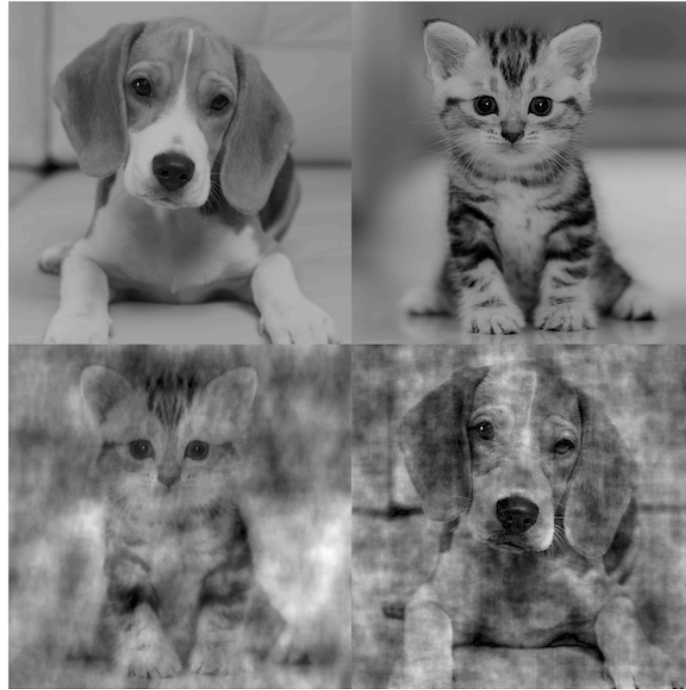


Figure 10.1: Importance of the phase of the FT. The two top images (dog and cat) are combined first by using the magnitude of the FT of the dog and the phase of the FT of the cat (bottom left image shows the results, resembling a cat). Similarly, the images are also combined by using the magnitude of the FT of the cat and the phase of the FT of the dog (bottom right image shows the results, resembling a dog).

Fourier domain. This formalism can help understand the effects of focused vs misfocused images, which can be described in terms of the OTF. Figure [10.2](#) shows an example of a focused vs misfocused image of the same object. The reason this example is interesting is because the object, a spoke pattern, naturally contains

10.4.1 Interactive exercise (Jupyter notebook)

Run this lecture's Jupyter notebook. Make sure to understand the connection between the OTF shape (as controlled by the interactive slider bar) and the nature of the artifacts in the measured image (including the region where the spokes disappear, and the region where the spokes are inverted. Take a look at the code (a few lines of code) and make sure you understand the overall computation (no need to worry about GUI details).

Question: Assuming that we have knowledge of the filtered image, as well as knowledge of the filter, can we undo the effect of the filter to recover the original spokes image? Does your answer depend on the choice of the OTF shape parameter as controlled by the slider?

Question: Suppose we want to sample a certain signal $f(x)$, which is band-limited with bandwidth B . Unfortunately, we only have the resources to sample with a spatial

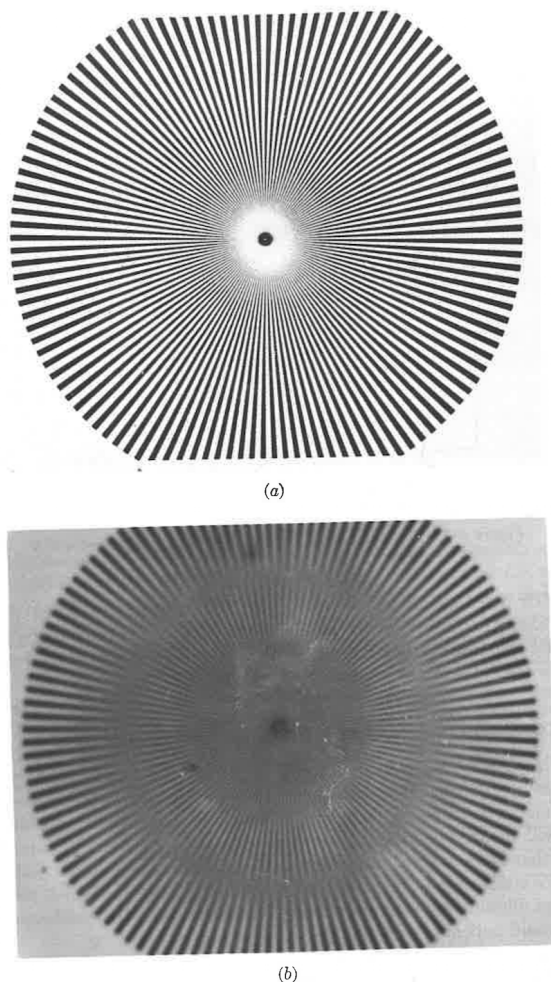


Figure 6-8 (a) Focused and (b) misfocused images of a spoke target.

from J.W. Goodman, Introduction to Fourier Optics, McGraw-Hill

Figure 10.2: Misfocused camera example, courtesy of Prof. Dwight Nishimura (Stanford University). The spoke pattern appears reasonably sharp throughout the entire image in the focused camera example (a). However, in the misfocused example (b), we can appreciate various artifacts at higher frequencies (near the center of the spoke pattern), including a ring where the spoke pattern disappears. Additionally, careful observation of the image suggests a frequency range where the response of the misfocused camera actually becomes negative.

spacing $\Delta x = 2\frac{1}{B}$, ie with a sampling frequency $\frac{1}{\Delta x} = \frac{B}{2}$. What can we do to our signal before we sample it, in order to avoid aliasing (even if we give up some information)?
Hint: think filtering.