

Lecture 15

Applications of the DFT: Image Reconstruction

15.1 Learning Objectives

- Apply the DFT to reconstruct medical images measured in Fourier space (e.g., MRI)
- Recognize the propagation of signal imperfections in the Fourier domain, which lead to artifacts in the image domain.

15.2 Introduction

Perhaps the most direct example application of the Fourier Transform in imaging is MRI image reconstruction. In MRI, data are sampled in Fourier space (termed k-space in MRI lingo). In other words, during an MRI acquisition we travel through k-space, collecting samples at various k-space locations as we go. When we have 'enough' samples, we typically perform an iFFT (in 2D or 3D, depending on the specific acquisition) in order to obtain our reconstructed image.

15.3 MRI Reconstruction

Reconstruction of MR images from MR raw data, which is acquired in the Fourier space (termed “k-space” in MRI lingo), can be as simple as a DFT in 2D (for 2D MR imaging, see illustration in Figure [15.1](#)) or a DFT in 3D (for 3D MR imaging).

Additionally, reconstruction of MR spectra (ie: in MR spectroscopy instead of MR imaging) from raw MR data is typically based on a 1D DFT. In other words, the DFT (and therefore the FFT) is pervasive in the reconstruction of images and spectra from MR data. In the next sections, we will cover several examples of data corruption and artifacts, which can be understood based on our study of Fourier Transforms and DFTs.

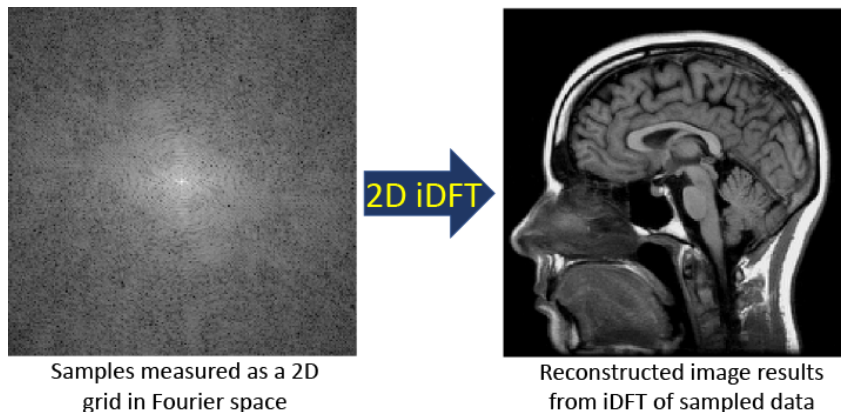


Figure 15.1: Basic image reconstruction in MRI.

15.4 Corrupted samples

15.4.1 Spike Noise

Occasionally when acquiring MRI data, a single k-space sample at some arbitrary k-space location (k_x, k_y) will be corrupted by a large error ('spike') of amplitude a :

$$\hat{f}_{\text{measured}}(k_x, k_y) = \hat{f}(k_x, k_y) + a\delta(k_x - k_{x,0}, k_y - k_{y,0}) \quad (15.1)$$

What will be the corresponding error in the reconstructed image?

15.4.2 Motion

Motion artifacts are a common challenge in MRI and other imaging modalities. In MRI, these artifacts can be easily replicated and understood via Fourier analysis. Motion artifacts in MRI are typically mild/negligible along one dimension (where the corresponding Fourier dimension is sampled very rapidly), whereas they can be severe along the other dimension(s) (where the corresponding Fourier dimension(s) are sampled slowly). This effect is illustrated in Figure [15.2](#) and can be replicated with some simple Matlab code, as shown below.

```
% Generate some true image and measure its dimensions
im0 = phantom(256);
[sx,sy] = size(im0);

% Generate some simulated physiological motion as a function of time
t = 1:sx;
mymotion = 2*sin(2*pi*0.1*t);

% Create motion-corrupted Fourier-space data based on the
% image and motion defined above
```

```

meas_imk = zeros(sx,sy);
for cur_t=1:length(t)
    cur_im0 = circshift(im0,[round(mymotion(cur_t)),round(mymotion(cur_t))]);
    cur_im0k = fftshift(fftn(ifftshift(cur_im0)));
    meas_imk(cur_t,:) = cur_im0k(cur_t,:);
end

% Reconstruct and show the true and motion-corrupted images
im = fftshift(ifftn(ifftshift(meas_imk)));
imagesc(abs(cat(2,im0,im)));axis equal tight off;colormap gray

```

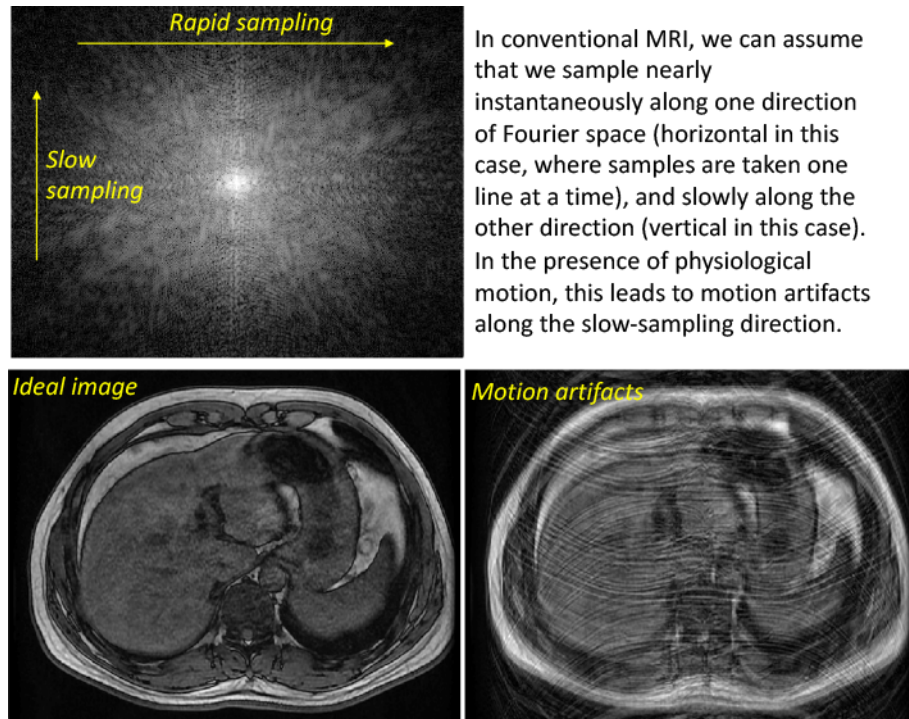


Figure 15.2: Physiological motion in MRI leads to motion artifacts along those directions that are sampled slowly. This effect is due to an inconsistency in the lines of Fourier space acquired at different stages of motion. In other words, different sampled lines in Fourier space correspond to images that are shifted with respect to each other. When we perform the inverse DFT (in 2D in this case) to reconstruct our image, these inconsistencies lead to deleterious motion artifacts observable in image space.

15.5 Missing samples

15.5.1 Artifacts from Missing Samples

Due to acquisition speed limitations and other challenges, in MRI we sometimes are not able to collect sufficient samples in Fourier space to obtain an adequate image. The resulting artifacts can be understood via Fourier analysis, as shown in Figure [15.3](#). Below is some Matlab code to reproduce this artifact:

```
% Generate some true image
im0 = phantom(256);

% Do 2D DFT
im0k = fftshift(fftn(ifftshift(im0)));

% Zero out every other line in DFT space
im1k = im0k;
im1k(2:2:end,:) = 0;

% Do 2D IDFT
im1 = fftshift(ifftn(ifftshift(im1k)));

% Display the original and reconstructed images
imagesc(abs(cat(2,im0,im1)));axis equal tight off;colormap gray
```

15.5.2 Advanced Image Reconstruction

There are a variety of advanced image reconstruction techniques that are often able to overcome the presence of missing samples in imaging. These techniques are based on including additional information into the reconstruction. This additional information can originate from various sources, including the presence of multiple receivers acquiring samples simultaneously, or prior information about image properties (eg: sparsity), or Machine Learning training using a large number of source data / reconstructed image pairs. In the optimization portion of this course, we will provide some examples of these advanced techniques.

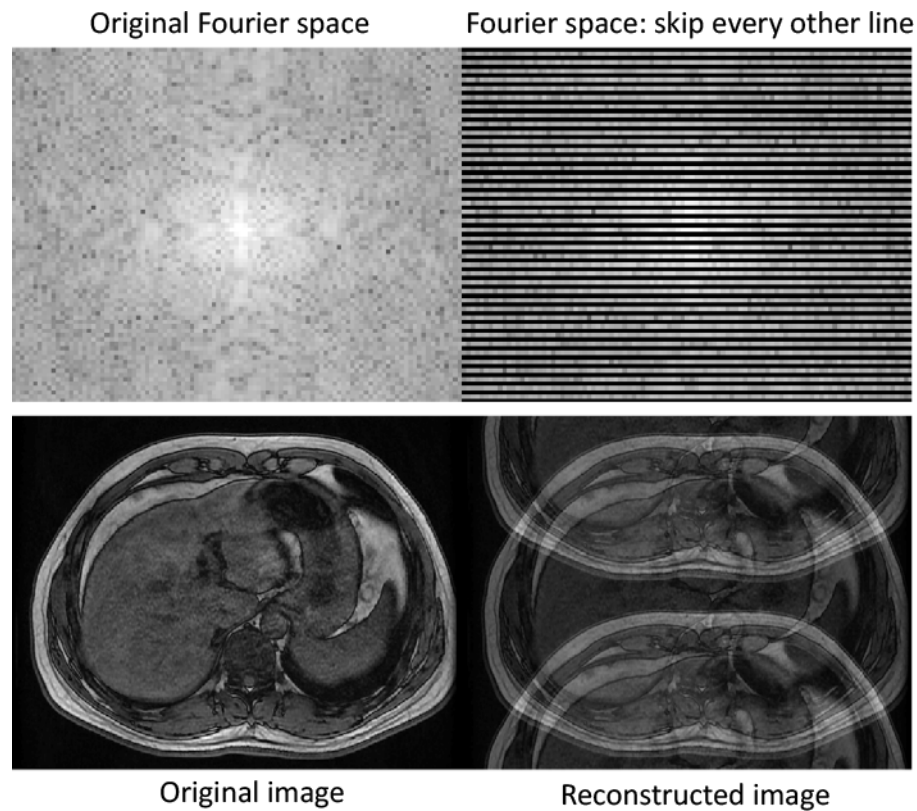


Figure 15.3: Missing data in Fourier space leads to artifacts in the reconstructed images. In the case where we skip every other line in Fourier space, the corresponding artifacts can be understood in terms of sampling and overlap (remember our sampling lectures!), and result in 2-fold overlap in our images.

