

Lecture 19

Wavelets in Multiple Dimensions

19.1 Learning Objectives

- Understand the filter bank interpretation of multi-resolution wavelet analysis in several dimensions.
- Recognize basic Matlab commands for multi-dimensional wavelet analysis and synthesis.
- Recognize example applications of multi-dimensional wavelet analysis in image compression and image reconstruction.

19.2 Calculation of the 2D DWT

In this lecture, we will focus on 2D DWT transforms, for simplicity. Further, we will focus on 2D transforms that are separable, ie: the filters in 2D can be decomposed as a filter along the horizontal dimension and another filter along the vertical dimension.

The computation of the DWT in 2D dimensions can be formulated as a filter bank, similarly to the 1D case. However, in the 2D case we need to apply filters along the horizontal and vertical dimensions of the image. These filters can be applied sequentially, as shown in Figure [19.1](#).

In Matlab, a single-level 2D wavelet decomposition can be performed with the command:

```
[cA, cH, cV, cD] = dwt2(f, wname);
```

where **f** is the input image, **wname** contains the name of the type of wavelets (eg: 'haar', 'db2', 'db3', ...), and **cA**, **cH**, **cV**, and **cD** contain the approximation coefficients (top left image), and detail coefficients along the horizontal, vertical, and diagonal directions respectively. A multi-level wavelet decomposition can be performed using the command **wavedec2**. The **dwt2** and **wavedec2** operations can be inverted to recover the original signal, using the commands **idwt2** and **waverec2**, respectively.

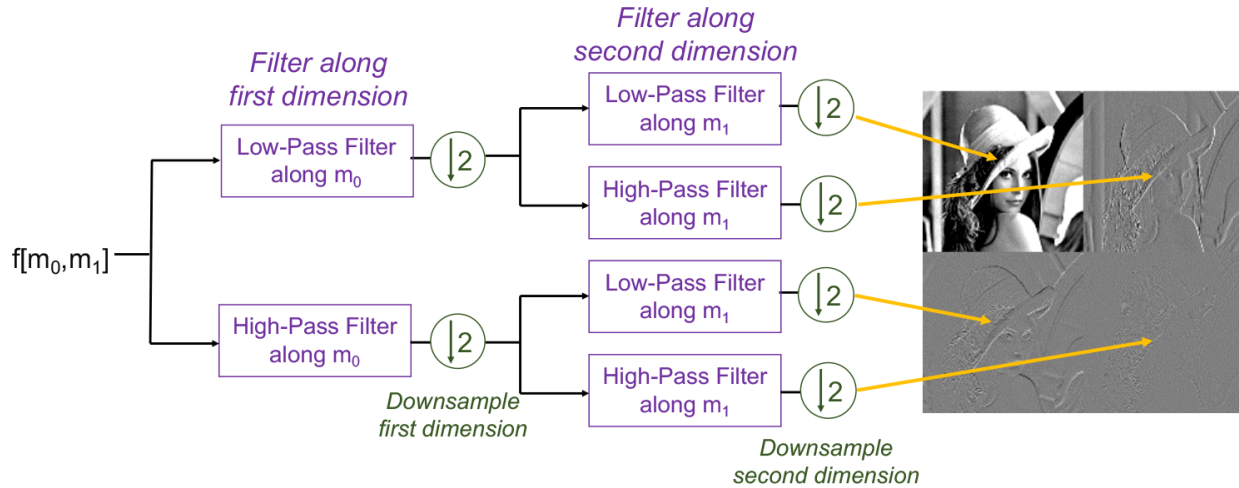


Figure 19.1: Single-Level decomposition of a 2D image, resulting in four different sets of coefficients. Additional levels of decomposition can be added for true multi-resolution analysis, by repeating the same scheme on the low-pass set of coefficients (top left corner of the image).

By applying the structure shown in figure [19.1](#) multiple times sequentially, we obtain multi-level decompositions, where the approximation coefficients at each level are fed as input to the next level. An example three-level decomposition of a brain MRI using Haar wavelets is shown in figure [19.2](#).

Haar wavelet decomposition of a brain MRI

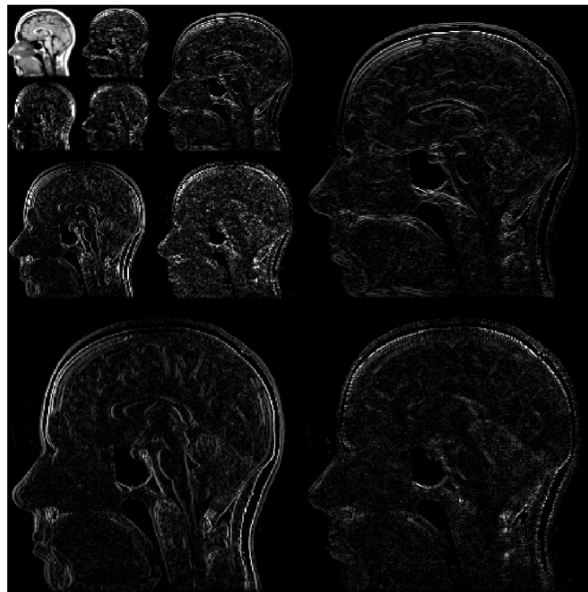


Figure 19.2: Three-level Haar wavelet decomposition of a brain MRI, performed using Matlab command `wavedec2`.

19.3 Application of the 2D DWT: Image Compression

In the previous lecture, we described an example of the use of wavelets for signal denoising. The same framework (eg: using soft thresholding) is directly applicable in 2D to image denoising.

For a different application, in this lecture we turn to image compression using wavelets. The original JPEG file format was based on the discrete cosine transform (DCT), a variation of the DFT. The newer JPEG 2000 format is based on wavelet decomposition (among other updates relative to the original JPEG). The use of wavelets leads to several advantages of JPEG 2000 over JPEG, including:

- *Improved compression ratio:* The wavelet-based JPEG 2000 can provide improved compression while maintaining image fidelity. This advantage is attributed to the use of the DWT and a more sophisticated entropy encoding scheme.
- *Multi-resolution representation:* This feature of wavelet analysis can in principle enable initially coarse, and then progressively finer rendering of an image as the data arrives.

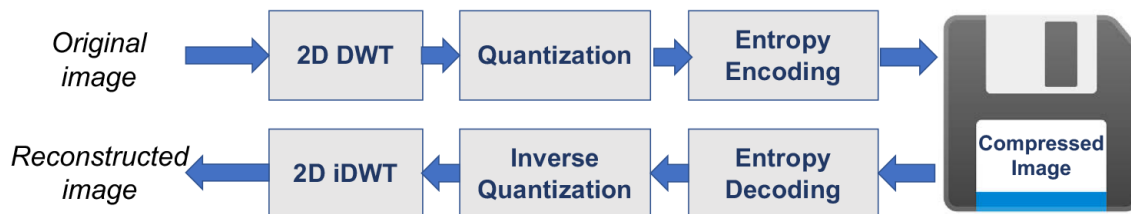


Figure 19.3: JPEG 2000 relies on a wavelet decomposition in order to ‘sparsify’ the representation of an image (ie: obtain a representation that has many zero or near-zero coefficients), which enables efficient compression. In the usual lossy JPEG format, a wavelet known as Daubechies 9/7 is used.

Although JPEG 2000 is still in use today, it is not nearly as widely available as the original JPEG format. This relative lack of success may be attributed to several factors, including technical complexity and the very widespread availability of the original JPEG.

19.4 Application of the 2D DWT: Image Reconstruction

An illustration of the use of the DWT for medical image reconstruction is shown in figure [19.4](#). Although we will not delve into the details of these advanced image reconstruction methods in this course (this will be covered in more detail in Med Physics 574), the bottomline is: the sparse representation that is provided by wavelet transforms can

be exploited for different purposes. In this case, this sparsity is exploited to enable reconstruction of high-quality MR images from undersampled Fourier-space (k-space in MRI lingo) data.

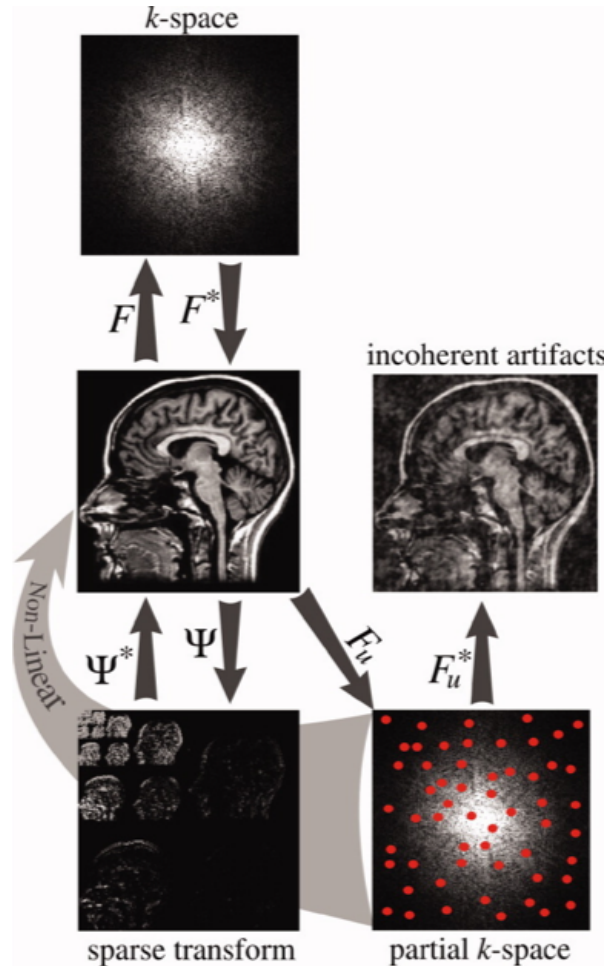


Figure 19.4: Image reconstruction is another area where wavelets have made an impact. The ability of wavelets to provide sparse representations of an image has been leveraged to enable improved reconstruction of medical images from undersampled data. This has been an area of active research for the past 10-15 years. For details, see (Lustig M et al, Sparse MRI: The application of compressed sensing for rapid MR imaging, Volume: 58, Issue: 6, Pages: 1182-1195, First published: 29 October 2007).